Mathematics for Management Online Course:
Algebra Section: Concept Summary

Algebra

Solving Linear Equations in One Variable
Manipulate the equation using Rule 1 so that all the terms involving the variable (call it \( x \)) are on one side of the equation and all constants are on the other side. Then use Rule 2 to solve for \( x \).

Rule 1: Adding the same quantity to both sides of an equation does not change the set of solutions to that equation.

Rule 2: Multiplying or dividing both sides of an equation by the same nonzero number does not change the set of solutions to that equation.

Straight Lines: Slope Intercept Form
A straight line with slope \( m \) and \( y \)-intercept \( (b, 0) \) has the equation \( y = mx + b \).

Point Slope Form of a Line Equation
Given two points on a line, \((x_0, y_0)\) and \((x_1, y_1)\), find the line's slope \( m = \frac{1-0}{1-0} \).

Then the equation of the line may be written as \( y - y_0 = m(x - x_0) \).

Solving Two Linear Equations
Two linear equations in two variables (call them \( x \) and \( y \)) have no solution, an infinite number of solutions, or a unique solution. You may solve two linear equations by either substitution or elimination.

- **Substitution**: Use one equation to solve for one variable in terms of the other (say, \( x \) in terms of \( y \)). Then substitute this relationship for each occurrence of \( x \) in the remaining equation. Now solve the remaining equation for \( y \). Given that you know \( x \) in terms of \( y \), you also know \( x \).

- **Elimination**: Add a multiple of one equation to the other equation to eliminate a variable (say, \( x \)) from the other equation. Solve the resulting equation for the remaining variable (\( y \)). Substitute this value of \( y \) in either of the original equations to find \( x \).

Linear Inequalities: One Variable
Use the following rules to solve for the set of values satisfying a linear inequality.

- If you add the same number to both sides of an inequality, the resulting inequality has the same direction as the original inequality. For example, if the original inequality were a "less than" inequality, the resulting inequality would also be a "less than" inequality.
• If you multiply or divide both sides of an inequality by the same positive number, the resulting inequality has the same direction as the original inequality. For example, if the original inequality were a "less than" inequality, the resulting inequality would also be a "less than" inequality.

• If you multiply or divide both sides of an inequality by the same negative number, the resulting inequality has the opposite direction as the original inequality. For example, if the original inequality were a "less than" inequality, the resulting inequality would be a "greater than" inequality.

**Linear Inequalities: Two Variables**

Change the inequality sign to an equals sign and graph the resulting line. The set of points satisfying the linear inequality will be all points on one side of the line (including the line). To find the side of the line that satisfies the inequality, simply choose a point (call it P) not on the line. If the point satisfies the inequality, then shade all points on P's side of the line; otherwise, shade all points not on the same side of the line as P.

**Parabolas**

The graph of \( y = ax^2 + bx + c \) is called a **parabola**. If \( a < 0 \), as \( x \) increases to the value \(-b/2a\), the value of the quadratic function will increase, and after \(-b/2a\) is reached, the value of the function will decrease. If \( a > 0 \), as \( x \) increases to the value of \(-b/2a\), the value of the quadratic function will decrease, and after \(-b/2a\) is reached, the value of the function will increase. Any parabola is symmetric about the line \( x = -b/2a \). That is, if \( x \) is \( k \) units larger than \(-b/2a\) and \( k \) units smaller than \(-b/2a\), the function has the same value for both these values of \( x \).

**The Quadratic Formula**

To solve for the values of \( x \) satisfying \( ax^2 + bx + c = 0 \), substitute \( a \), \( b \), and \( c \) into the following equation, called the **quadratic formula**:

\[
 x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

**Exponents**

An **exponent** is shorthand for repeated multiplication. For example, \( 2^4 = 2 \times 2 \times 2 \times 2 \). The following rules apply to exponents:
Rule 1: $x^0 = 1$

Rule 2: When multiplying like terms involving exponents, add the exponents. That is, $x^a \times x^b = x^{a+b}$.

Rule 3: $(xy)^a = x^a \times y^a$.

Rule 4: $(x^a)^b = x^{ab}$.

Rule 5: $(x/y)^a = x^{a}/y^a$.

Rule 6: $x^{-a} = 1/x^a$.

Rule 7: $=x^{a-b}$.

**Power Function**

The function $y = ax^b$ is called a **power function**. Usually in business, $a > 0$ and $x > 0$. Then for $b > 1$, as $x$ increases, $y$ increases and the graph gets steeper. For $0 < b < 1$, as $x$ increases, $y$ increases but the graph gets flatter. For $b < 0$, as $x$ increases, $y$ decreases and the graph gets flatter.

**Cobb Douglas Function**

If $K = \text{capital}$, $L = \text{labor}$, and $0 < a < 1$, the output of an organization or economy is often modeled by the Cobb-Douglas production function, which is of the form $f(K, L) = K^aL^{1-a}$.

**Order of Operations**

In the evaluation of mathematical expressions, the order of operations is as follows:

- P (Parentheses): If the expression contains parentheses, first evaluate all expressions within parentheses, working from the innermost set of parentheses out.

- E (Exponents): Next, perform all operations involving exponents.

- MD (Multiplication and Division): Next, perform all multiplication and division calculations from left to right.

- AS (Addition and Subtraction): Finally, perform all addition and subtraction calculations from left to right.

This hierarchy is easily remembered by using the pneumonic device PEMDAS, or Please Excuse My Dear Aunt Sally.

**Entering and Graphing Functions in Excel**

- All Excel formulas begin with an = sign.

- Use the ^ symbol to raise a number to a power and * for multiplication.

- Excel follows PEMDAS.
• Use the Scatter option from the Insert tab of the Excel ribbon to graph a function.

**Inverse Function**
If \( y = f(x) \), the inverse function of \( f \) (call it \( g \)) is found by solving for \( x \) in terms of \( y \). The expression is often written as \( x = g(y) \).

**Ratio**
The ratio of two numbers reflects their relative sizes. For example, if you want to divide an inheritance between two siblings in the ratio 2:3, the first sibling will get 2/3 as much as the second sibling or, equivalently, the second sibling will get 3/2 as much as the first sibling.

**Percentage**
A percentage is simply mathematical shorthand for one hundredth. For example, if a bookstore marks up the price of a book 40% over the wholesale price, you can compute the retail price by adding forty hundredths, or 0.4 times, the wholesale price to the wholesale price.

**Elasticity of Demand**
The demand elasticity (\( E \)) for a product is the percentage change in demand that results from a 1% increase in the product's price. If \( E < -1 \), the demand is elastic; if \( -1 < E < 0 \), the demand is inelastic.

**Logarithm**
- Assuming a positive number \( b \), the logarithm of a number \( x \) to the base \( b \) is the power to which \( b \) must be raised to result in the number \( x \).
- If you write \( \log_b x = c \), then \( b^c = x \).
- When \( b = e \) (\( e \) is approximately 2.7182), write \( \ln x \) instead of \( \log_e x \).
- The Excel function \( \text{LOG}(x,b) \) returns \( \log_b x \).
- The following rules apply to logarithms:
  
  **Rule 1**: \( \log_b x + \log_b y = \log_b xy \).
  
  **Rule 2**: \( \log_b x - \log_b y = \log_b x/y \).
  
  **Rule 3**: \( \log_b x^c = c \log_b x \).

**Index Numbers**
An index number indicates the percentage change in a quantity, relative to a base level that is assigned a value of 100. For example, suppose that the base year for GNP is 2000 and that the GNP in 2000 is $4 trillion. If the GNP in 2008 is $6 trillion, the 2008 GNP index is 150.